Vector Geometry

Fact —

- $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ ("end minus start", with position vectors from O).
- For a column vector, $\left|\begin{pmatrix} p \\ q \end{pmatrix}\right| = \sqrt{p^2 + q^2}$.
- Two vectors are **parallel** exactly when one is a scalar multiple of the other.
- A, B, C are **collinear** when $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ (they already share B).
- Midpoint of AB: position vector $\frac{1}{2}(\mathbf{a} + \mathbf{b})$.

Example

$\mathbf{u} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 6 \\ k \end{pmatrix}$.

1. Find $|\mathbf{u}|$.
2. Find k such that $\mathbf{v}$ is parallel to $\begin{pmatrix} 9 \\ 12 \end{pmatrix}$.
3. Find both values of k such that $|\mathbf{v}| = 10$.

Example

Relative to an origin O, the points A, B, C have position vectors

$$\mathbf{a} + 2\mathbf{b}, \qquad 3\mathbf{a} + 3\mathbf{b}, \qquad 7\mathbf{a} + 5\mathbf{b}$$

where $\mathbf{a}$ and $\mathbf{b}$ are non-parallel vectors. Prove that A, B and C are collinear, and state the ratio $AB : BC$.

Example

$OACB$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of AC and N is the point on BC with $BN : NC = 3 : 1$. Express in terms of $\mathbf{a}$ and $\mathbf{b}$:

1. $\overrightarrow{OC}$
2. $\overrightarrow{OM}$
3. $\overrightarrow{MN}$

Textbook Exercises: SPS Course 2.6, Exercises 4A and 4B

Parameters

When a point lies on a line but its position is unknown, introduce a parameter: X on the line through B and M means $\overrightarrow{BX} = \mu\overrightarrow{BM}$ for some scalar μ.

Fact — If $\mathbf{a}$ and $\mathbf{b}$ are non-parallel and

$$\lambda_1\mathbf{a} + \mu_1\mathbf{b} = \lambda_2\mathbf{a} + \mu_2\mathbf{b},$$

then $\lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$: coefficients can be equated.

Example

In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of OA, and N lies on AB with $AN : NB = 1 : 2$. The lines ON and BM intersect at X.
Find $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, and hence the ratio $OX : XN$.

Example

$OACB$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. D is the midpoint of AC. The line OD is extended to meet the line BC extended at E.
Find $\overrightarrow{OE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, and show that C is the midpoint of BE.

Textbook Exercises: SPS Course 2.6, Exercises 4C and 4D